Ch 10 Exponential and Logarithmic Functions

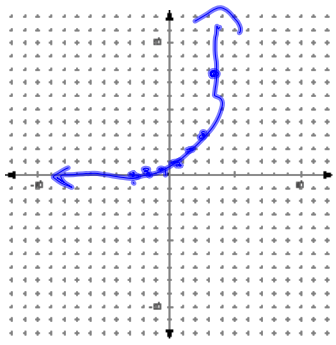
10-1 Exponential Functions

Exponential function--function with a variable in the exponent

ex

$y = 2^x$

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8



Form

$y = a\,b^x$

$a \neq 0$

$b > 0$

$b \neq 1$

graph on calculator

Characteristics (p524)

1. The function is continuous and one-to-one.
2. The domain is the set of all real numbers.
3. The x-axis is an asymptote of the graph.
4. The range is the set of all positive numbers if $a > 0$ and all negative numbers if $a < 0$.
5. The graph contains the point $(0, a)$. That is, the y-intercept is a.
6. The graphs of $y = ab^x$ and $y = a\left(\frac{1}{b}\right)^x$ are reflections across the y-axis.

Write an exponential function whose graph passes through the given points.
(0, 5) (2, 45)

$y = 5 \cdot 3^x$

$y = a \cdot b^x$

$5 = a \cdot b^0$

$5 = a \cdot 1$

$y = 5b^x$

$45 = 5b^2$

$9 = b^2$

$\pm 3 = b$

$3 = b$

Write an exponential function whose graph passes through the given points.
(0, 4) (3, ½)

$a = 4$

$y = 4b^x$

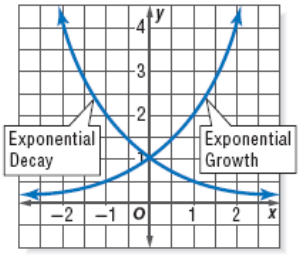
$\frac{1}{2} = 4 \cdot b^3$

$\left(\frac{1}{8} = b^3\right)^{1/3}$

$\frac{1}{2} = b$

$y = 4\left(\frac{1}{2}\right)^x$

There are two types of exponential functions: **exponential growth** and **exponential decay**. The base of an exponential growth function is a number greater than one. The base of an exponential decay function is a number between 0 and 1.



Key Concept — *Exponential Growth and Decay*

- If $a > 0$ and $b > 1$, the function $y = ab^x$ represents exponential growth.
- If $a > 0$ and $0 < b < 1$, the function $y = ab^x$ represents exponential decay.

FARMING In 1983, there were 102,000 farms in Minnesota, but by 1998, this number had dropped to 80,000.

a. Write an exponential function of the form $y = ab^x$ that could be used to model the farm population y of Minnesota. Write the function in terms of x, the number of years since 1983.

$y = 102{,}000(.98)^x$ $y = ab^x$

$y = 102{,}000\,b^x$

a - original amt.
b = Rate of change
y = new amt.
x = time

$80{,}000 = 102{,}000\,b^{15}$

$(.78)^{\frac{1}{15}} = b^{15}$

$.98 = b$

b. Suppose the number of farms in Minnesota continues to decline at the same rate. Estimate the number of farms in 2010.

$y = 102{,}000(.98)^{27}$

$\approx 59{,}000$

In December of 1990, there were 5,283,000 cell phones in the U.S. In December of 2000, there were 109,478,000.

Write an equation in exponential form. Find the number of cell phones in 2012.

$y = 5{,}283{,}000\,b^x$ $109{,}478{,}000 = 5{,}283{,}000\,b^{10}$

$1.35 = b$

Solving exponential equations.

ex
$8^x = \frac{1}{4}$

$8^{-\frac{2}{3}}$ ✓

$2^{3x} = 2^{-2}$

$3x = -2$

$x = -\frac{2}{3}$ check

ex

$5^{4-t} = 25^{t-1}$

$5^{4-t} = 5^{2(t-1)}$

$4-t = 2t-2$

$6 = 3t$

$2 = t$ Check: ✓

ex $32 = 2^5$

$\sqrt[3]{32} = 2^x$

$2^{5/3} = 2^x$

$\frac{5}{3} = x$ ✓

ex

$9^{x-3} = 27$

$(3^2)^{(x-3)} = 3^3$

$2x-6 = 3$

$2x = 9$

$x = 4.5$ ✓

Do:

1. $4^{x-2} = 64^x$

2. $4^{2x+5} = 16^{x+1}$

Exponential Inequalities

ex

$3^x < \frac{1}{27}$

$27 = 3^3$

$9 \quad 3$

$3 \; 3$

$3^x < 3^{-3}$

$x < -3$

ex

$5^x > \sqrt{125}$

$5^x > \sqrt{5^3}$

$5^x > 5^{3/2}$

$x > \frac{3}{2}$

HW
p528-529
21, 25, 27-29, 33, 35, 39-53odd,
57-60(look at picture)